

THE MATHEMATICAL ANALYSIS OF GEORGIAN DECLARATIVE VERBS

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**Abstract.** The first steps with the purpose of studying the complex lingual-logical nature of Georgian Declarative Verbs (GDV) from the mathematical point of view had been taken by Doctor of Science K.Pkhakadze<sup>1</sup> [1]-[3], [5]-[14]. It must be underlined that in the fundamental monographic work “On the Linguistic Relations and Logical Declension in Georgian” [14], which was published in 2005, K.Pkhakadze describes the Core Part (CP) of Georgian Language and Thinking (GLT) (CPofGLT)<sup>2</sup> as a mathematical theory. On the base of K. Pkhakadze’s the above mentioned researches there was brought to light the new lingual ideology, which we call as the Georgian Lingual Ideology (GLI), because of it is in the fully accordance to the nature of the GLT. Besides the above mentioned, the GLI is fundamentally based on Prof. Sh. Pkhakadze’s Notation Theory and on his General Semantic Program. The aim of this article is to draw out the theoretical bases and expected results of L. Abzianidze’s and A. Maskharashvili’s Master’s Theses (MTs), which general title is “The Mathematical Analysis Of Georgian Declarative Verbs and the First Version of The Intellectual Computer System in Georgian Written Language”. In these, there are planned to make almost complete mathematical description of the Georgian Declarative Verbs (GDVs) and to study their mathematical properties: this will make us able to make almost complete Euclid type foundation of CPofGLT, toward which we are aimed and which as possibility is declared by the above mentioned GLI.

**Keywords and phrases:** Logical Grammar of the Georgian Language, Mathematical Analysis of Georgian Declarative Verbs and Sentences, Georgian Lingual Ideology, Georgian Mathematical Language.

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**1. The basic part.** In the [2] it was firstly declared that in any Georgian declarative sentence a noun is understood either as a set, which is naturally defined in thought by this noun, or as a variable (i.e. universal constant), or as a constant (i.e. existential constant) defined on this set. For example, in the thought the Georgian

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<sup>2</sup>Sometimes we call the Georgian language and thinking as Georgian Lingual Theory (GLT). This means that the readers can read the CPofGLT as the core the part of Georgian language and thinking, as well as the core part Georgian lingual theory.

noun 'adamiani $\approx$ human'<sup>3</sup> is understood as the set of all humans, which we denote as '{adamiani}={human}'.

Also, in the [1] we have already proved that in the Georgian sentence 'adamiani mokvdavia  $\approx$  human is mortal' the word 'adamiani  $\approx$  human' is understood as a variable defined on the set {adamiani}, while in the sentence 'adamiani mokvda  $\approx$  human died' the same word is understood as a constant defined on the same set.

Besides the above mentioned, in [14] we have already proved that the full stop, i.e. the ending sign '.' of the GDSs is the left 1-place contracting symbol defined as follow:

S. \_\_\_\_\_[[S] = t], where:

1. the area of definition of the meta-variable S is a set of all formal Declarative Sentences (DSs), i.e. a set of all well formed expressions, which together with the full stop forms natural, i.e. semantic DSs<sup>4</sup>.

2. The composed sign '= t' is the left 1-place interpreter operator of formal declarative sentences, which is obtained by inserting t in the right place of the left and the right 1-place interpreter operator '='.

Hence, according to the GLI in the Georgian thinking language, which we call also as Georgian mathematical language, any 'S.' DS is understood as a lingual-mathematical expression '[[S]=t]': this underlines that any DS is already interpreted by the speaker person as a true fact<sup>5</sup>.

In the [14] it is already proved that any Declarative Verb (DV) [ $V^n(\text{GN}\alpha_1, \text{GN}\alpha_2, \dots, \text{GN}\alpha_n)$ ] proposed in the Declarative Mood (DM) in the Georgian thinking language forms general DS<sup>6</sup> of the [ $V^n(\text{GN}\alpha_1, \text{GN}\alpha_2, \dots, \text{GN}\alpha_n)=t$ ] type, where:

1. V is a word, which expresses this DV;
2. The upper index n indicates the placeness of this V;
3.  $\text{GN}\alpha_k$  ( $k \in \{1, 2, \dots, n\}$ ) is either a personal pronoun, or a general pronoun, which is in the one of the declination forms from the following 14 declination forms<sup>7</sup>: Ni, Nis, Nisken, Nisgan, Nist'vis, Nidan, Nit', Ns, Nshi, Nze, Nad, Namde, Nt'an, Nma.
4. '=t', as was already mentioned, is an interpreter operator of formal DSs, which, sometimes we call also as a logical operator of the DM.

All above mentioned give us opportunity to make the natural definition of the Georgian DSs (resp. DVs): namely, according to Logical Grammar of the Georgian Language (LGofGL) an expression (resp. a word) of Georgian written language is a DS

<sup>3</sup>The sign  $\approx$  indicates us on that very clear fact that lexical "identity" of the words 'human' and 'adamiani' dose not implies the identity of this word in the Georgian and English thought languages, i.e in the Georgian and English thinking languages.

<sup>4</sup>We make difference between formal and semantic DSs. The formal DS 'The apple is red' gives no information. The semantic, i.e. the natural DS 'The apple is red' dictates that for speaker person (apple) $\in$ {red} is true and (apple) $\in$ { red}) is false.

<sup>5</sup>This does not mean that the fact, given by this declarative sentence, is really true one. Also, this underlined that it is impossibility to solve completely even the problem of the simple DS in the natural languages without basic logical categories, such are the truth values: 't' and 'f' and the left and the right 1-place interpreter operator '='.

<sup>6</sup>By the utterance the DV 'vsvav  $\approx$  Lam\_drinking' in DM there is formed general DS '[(me) [vsvav<sup>2</sup> (me, Ns)] (ragacas)]=t  $\approx$  [(I) [Lam\_drinking<sup>2</sup>(I, N)] (something)]=t' general DS.

<sup>7</sup>The mathematical analyze of the Georgian language and thinking, which was made by K. Pkhakadze in the [14], there was proved that in Georgian language there are 14 different declination forms, i.e. cases.

(resp. DV) if and only if by its uttering in DM there is obtained such type semantic understanding about which to ask the question: “is it true or false?” - is sensible<sup>8</sup>.

Moreover, in the [14] it is already sufficiently proved that:

1. The placeness of the DVs and their allowable specification rules, i.e. the semantic and the syntactic areas of definition of their places are univocally definable by these verbs;
2. Any simple DS of the Georgian language is a result of some allowable specification of that DV on the basis of which it is formed.

Hence, we can conclude that any DV, which is a sentence itself, at the same time, is the most general sentence built by this DV, and every other sentence built by this DV is obtained by the different allowable specifications of this most general sentence.

This makes clear that in LGofGL the set of DVs can be considered as the basis set of this theory. It means that any statement of this theory is obtained through the operations on the elements of this basis set. Also, this makes clear that the basic operation in LGofGL is the operation of replacement of a noun phrase (phrases) by a noun phrase (phrases).

Below, in order to give more argumentations for the above mentioned insight, we will consider some aspects of the logical negation of the GDVs: as was already mentioned, in the Georgian language DVs have the following complex lingual-logical form:  $[V^n(GN\alpha_1, GN\alpha_2, \dots, GN\alpha_n)] = t$ <sup>9</sup>. This means that, in order to make its logical negation, we must logically negate its main, i.e. head operator, which is ‘= t’:

$$\neg[V^n(GN\alpha_1, GN\alpha_2, \dots, GN\alpha_n) = t] \Leftrightarrow [V^n(GN\alpha_1, GN\alpha_2, \dots, GN\alpha_n) \neq t] \Leftrightarrow \\ \Leftrightarrow [V^n(GN\alpha_1, GN\alpha_2, \dots, GN\alpha_n) = f] \Leftrightarrow [arV^n(GN\alpha_1, GN\alpha_2, \dots, GN\alpha_n) = t]$$

Above, ‘ar ≈ not’ is the right 1-place lingual-logical operator of negation, the lingual area of definition of which consists all GDVs, but, as it was seen, it operates logically on ‘=t ‘ and as a results gives ‘=f ‘. An example:

$$\neg[[is][unda](cavides)] = t \Leftrightarrow [[is][ar[unda]](cavides)] = t \Leftrightarrow \Leftrightarrow \\ [[is][unda](cavides)] = f \text{ (1G)} \quad \neg[[he][must](leave)] = t \Leftrightarrow \\ [[he][not[must]](leave)] = t \Leftrightarrow \Leftrightarrow [[he][must](leave)] = t \text{ (1E)}$$

Now let us one more time consider above already partially considered examples, where the noun ‘adamiani ≈ human’ is understood differently in different sentences: it is clear that the sentence ‘adamiani mokvda ≈ human died’ give us information about the particular human and not about all of them. This means that in this sentence ‘adamiani’ is understood as a constant, i.e. as an existential constant, the area of definition of which is the set {adamiani}. But, the sentence ‘adamiani mokvdavia ≈ human is mortal’ gives us information about all humans. This means that in this sentence ‘adamiani’ is understood as a variable, i.e., as a universal constant, the area of definition of which is the set {adamiani}. These circumstances point out that together with syntactic and semantic descriptions of the places of the verbs it is necessary to make logical type description of their places. For example: mokvda<sup>1</sup>(Ni) (resp.

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<sup>8</sup>By the utterance ‘the red apple’ in DM there is naturally obtained such type semantic understanding about which to ask the question: “is it true or false?” - is not sensible, when by the utterance ‘the apple is red’ in DM there is naturally obtained such type semantic understanding about which to ask the question: “is it true or false?” - is natural, i.e. is sensible.

<sup>9</sup>Sometimes, for simplicity, we do not use some pairs of parentheses.

$\text{mokvdavia}^1(\text{Ni})$ ) is a result of syntactic and semantic descriptions of the verb ‘mokvda’ (resp. ‘mokvdavia’). So, syntactically these verbs do not differ from each other. The following:

$\text{Ni} \in \{\text{mortality}\}:\text{mokvda}^1(\text{Ni})$  (resp.  $\text{Ni} \in \{\text{mortality}\}:\text{mokvdavia}^1(\text{Ni})$ ) is a result of syntactic and semantic type description of the verb ‘mokvda’ (resp. ‘mokvdavia’). So, general syntactic and semantic type of these verbs do not differ them.

$$\begin{aligned} &{}^c\text{Ni} \in \{\text{mortality}\}:[\text{mokvda}^1[(^c\text{Ni})]=\text{t}], \text{ i.e. } ({}^{\exists}\text{Ni}) \in \{\text{mortality}\}:[\text{mokvda}^1[({}^{\exists}\text{Ni})]=\text{t}] \\ &\text{(resp. } (^V\text{Ni}) \in \{\text{mortality}\}:[\text{mokvdavia}^1[(^V\text{Ni})]=\text{t}], \text{ i.e.} \\ &\quad ({}^{\forall}\text{Ni}) \in \{\text{mortality}\}:[\text{mokvdavia}^1[({}^{\forall}\text{Ni})]=\text{t}]) \end{aligned}$$

is a result of syntactic, semantic and logical type description of the verb ‘mokvda’ (resp. ‘mokvdavia’). So, by the general syntactic, semantic and logical type descriptions, i.e. by general mathematical description these verbs differ clearly from each other.

Now, it must be clear for the readers knowing Georgian language, that according to above shortly presented approaches the sentence ‘studenti kitxulobs cigns  $\approx$  a student reads a book’ represents following prefix quantifier formula with restricted quantifiers:

$$\exists(\text{studenti}) \in \{\text{studenti}\} \exists(\text{cigns}) \in \{\text{cigns}\}: [\text{kitxulobs}^2((\text{studenti}); (\text{cigns})) = \text{t}] \quad (2G)$$

$$\exists(\text{student}) \in \{\text{student}\} \exists(\text{book}) \in \{\text{book}\}: [\text{is\_reading}^2((\text{student}); (\text{book})) = \text{t}] \quad (2E)$$

which, in the infix quantifier form [4] will be written as follows:

$$[\text{kitxulobs}(({}^{\exists}\text{studenti}); ({}^{\exists}\text{cigns})) = \text{t}] \approx [\text{is\_reading}(({}^{\exists}\text{a\_student}); ({}^{\exists}\text{a\_book})) = \text{t}] \quad (3)$$

Also, according to the above described approaches the sentence ‘kvela studenti kitxulobs cigns  $\approx$  Every student reads a book’ in the infix quantifier form will be written as follows:

$$[\text{kitxulobs}(({}^{\forall}\text{studenti}); ({}^{\exists<\forall>}\text{cigns})) = \text{t}] \approx [\text{reads}(({}^{\forall}\text{student}); ({}^{\exists<\forall>}\text{book})) = \text{t}] \quad (4)$$

which, in the prefix quantifier form will be rewritten as follows:

$$\forall(\text{studenti}) \in \{\text{studenti}\} \exists(\text{cigns}) \in \{\text{cigns}\}: [\text{kitxulobs}((\text{studenti}); (\text{cigns})) = \text{t}] \quad (4G)$$

$$\forall(\text{student}) \in \{\text{student}\} \exists(\text{book}) \in \{\text{book}\}: [\text{is\_reading}((\text{student}); (\text{book})) = \text{t}] \quad (4E)$$

which semantic meaning, and it must be clear for readers knowing Georgian language, coincide with natural semantic meaning of (4) sentence.

This makes clear that in (4) existential constant ‘cigns  $\approx$  book’, which is inserted into the  $(-s)$  of the  $[\text{kitxulobs}(-i; -s)]$ , is in the operational scope of quantifier phrase ‘kvela studenti  $\approx$  every student’, which is inserted  $(-i)$  type place of  $[\text{kitxulobs}(-i; -s)]$ . The additional argumentation of the above mentioned is the following well known fact: despite of permutation of the words ‘kvela studenti  $\approx$  every student’ and ‘cigns  $\approx$  book’ in the sentence (4) the semantic meaning of it does not change<sup>10</sup>.

The above analysis proves the existence of premeditated harsh prefix constructions of restricted quantifiers with the DV ‘kitxulobs  $\approx$  is\_reading’, i.e. with the lingual-logical relation ‘kitxulobs $(-i; -s) \approx$  is\_reading  $(-i; -s)$ ’. So, according to our approaches  $\forall(\text{N}_i) \in \{\text{N}_i\} \exists(\text{N}_s) \in \{\text{N}_s\}: [\text{kitxulobs}(\text{N}_i; \text{N}_s)]$  is natural counterpart of the DV ‘kitxulobs  $\approx$  is\_reading’ in the GTL.

The same situation we have with the DV ‘caukitxia  $\approx$  he/she\_have\_red’, the natural counterpart of which, in the GLT, is  $\forall(\text{N}_s) \in \{\text{N}_s\} \exists(\text{N}_i) \in \{\text{N}_s\}: [\text{caukitxia}(\text{N}_s; \text{N}_i)]$ .<sup>11</sup>

<sup>10</sup>In the Georgian language ‘cigns kitxulobs kvela studenti = kvela studenti kitxulobs cigns  $\approx$  Every student reads a book’

<sup>11</sup>In the sentences ‘kvela students caukitxia cigni  $\approx$  every student he/she\_have\_red a book’ and ‘cigni caukitxia kvela students  $\approx$  a book he/she\_have\_red every student’ the simple noun ‘cigni  $\approx$  book’,

But, in Georgian there are the verbs which are not expressing the actions analogously already overviewed: let us consider the GDV ‘binadronben  $\approx$  live\_in’, which syntactically is described as ‘binadronben( $N_{ebi}, N_{si}$ )  $\approx$  live\_in (Ns, N)’ and the sentences (5) and (6):

‘aqlemebi udabnosi binadronben  $\approx$  camels live in desert’ (5)

‘udabnosi aqlemebi binadronben  $\approx$  in desert camels live in’ (6)

For readers knowing Georgian it is clear that none contextual semantic meaning of (5) and (6) sentences differ from each other and they coincide respectively with none contextual semantic meaning of (5.1) and (6.1) sentences:

‘kvela aqlemi udabnosi binadrobbs  $\approx$  all camels live in desert’ (5.1)

‘kvela udabnosi aqlemebi binadronben  $\approx$  in all deserts camels live in’ (6.1)

This means, that GDV ‘binadronben  $\approx$  live\_in’ is not syntactically and semantically ambiguous, but logically it is ambiguous: this means, that it is not characterized by uniquely premeditated harsh prefix constructions of restricted quantifiers and that ‘binadronben $\approx$ live\_in’ has two different logical descriptions.

**2. Conclusion.** Now, about the researches, which are planned in the L. Abzianidze’s and A. Maskharashvili’s MTs: on the base of above shortly described results and approaches, we claim that all simple GDSs are atomic formulas with infix quantifiers of above partially overviewed type. It is clear, that to prove here mentioned it is necessary to make mathematical description of the GDVs: it is clear, that without such type of descriptions of GDVs it will be impossible to describe the different type right, i.e. t-type and wrong, i.e. f-type inferences for Georgian thinking language: for example, it is clear that without logical description of ‘mokvda’ and ‘mokvdavia’, which are [mokvda<sup>1</sup>[(<sup>c</sup>Ni)]=t] and [mokvdavia<sup>1</sup>[(<sup>v</sup>Ni)]=t], it is impossible to declare (I) as the right, i.e. as a t-inference, and (II) as wrong, i.e. f-inference in the GLT.

$$\frac{[\text{mokvdavia}({}^vN_i)]=t}{X_i \text{ aris } N_i} \quad t \quad \text{(I)} \qquad \frac{[\text{mokvda}({}^cN_i)]=t}{[X_i \text{ aris } N_i]=t} \quad f \quad \text{(II)}$$

$$\frac{}{[\text{mokvdavia}(X_i)]=t} \qquad \frac{}{[\text{mokvda}(X_i)]=t}$$

We have to say, that above underlined scientific aims of the above mentioned MTs have fundamental scientific importance: because the results obtained with the researches planed by these MTs will help us to look at the mathematical nature of the Georgian thinking language, i.e. the Georgian Mathematical Language deeply, and this will help us to develop the rigorous mathematical researches in the Georgian thinking language with the aim of construction mathematical theory of the natural Georgian Language and thinking. In other words, the results obtained with the researches planned by these MTs will be a very important step made to the complete Euclid type foundation of the natural GLT.

R E F E R E N C E S

which is inserted into a place of the type (<sub>-i</sub>) of the [caukitxia(<sub>-s</sub> ; <sub>-i</sub>)], is in the operational scope of quantifier noun phrase ‘kvela students  $\approx$  every student’, which is inserted into a place of the type (<sub>-s</sub>) of the [caukitxia(<sub>-s</sub> ; <sub>-i</sub>)].

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